Name_____

This is Harold Kaplan's final project in SM162. It is handed out 4 April 2005. It is due back 19 April 2005, in class on paper, not e-mailed. A final project is like a final examination: it must be completed without help from anybody except Harold Kaplan. Kaplan will help in class, but not in extra instruction. You may use your own notes and any commercially prepared materials you choose. For each problem, hand in printouts (or pencillings) of program and answers. For each problem mentioning the word "error," explain why you think your error is as small as required. Each answer ought to go on the same paper as its program. The programs you write ought to have blank lines and indentation to show structure. Use `formatlong()` in the Lua programs. Use the "ax" on each program, so it has no lines but those necessary. Using calculus is permitted.

1 Use the definition of natural logarithm in our syllabus to write and run a Lua program to find the natural logarithms of 3, 4, and 12, with error no more than 1e-3. Have the program check to see how close the logarithm of 12 is to the sum of the logarithms of 3 and 4.

2 A probabilist needs $\int_1^\infty \exp(-x^{1.3})dx$, with an error no more than 1e-4. Write and run a Lua program for her purpose. (Hint: $\exp(-x^{1.3})$ is bounded by $\exp(-x)$ on her domain.

3 A sociologist needs to solve the differential equation $\frac{dy}{dx} = \sqrt{x} + \sqrt{y}$ starting at $x = 0$ and $y = 0$ and stopping at $x = 1$. She says the error must be no larger than one digit in the fourth decimal place, and also she needs a graph. Write and run an appropriate Lua program.

4 An economist assures me that she is going to use the series $\sum_{n=0}^{\infty} \frac{x^n}{n!n!n!}$ only for values of x between -1/2 and 1/2. She says she must have no more error than 1/1000. Write for her a Lua function that will add up enough terms of her series. (Maybe even you ought to design the function so it will give a diagnostic and quit if the argument is out of domain.) Then test the function by plugging in $x = 1/3$ and printing the answer.

Hand this question sheet in with the answer sheets.